equator, very much smaller than the object named, and (as I have endeavoured to prove in *Monthly Notices*, January 1904, p. 241) 25°, or 45<sup>m</sup>, east of it.

I must confess myself surprised at several of the statements made by Professor Hough, but will content myself with particular reference to one. He says (Monthly Notices, 1904, p. 552) that my "suggestion that discordant observations may be due to oscillation in position of the spots on Jupiter and Saturn is not sound." Now Professor Hough himself has proved by micrometric measures the oscillation of the spots on Jupiter! In his annual report of the Dearborn Observatory, 1882, p. 50, he distinctly remarks of a large white spot on Jupiter that, "observed continuously for a period of 252 days, it indicated sudden deviations in its apparent place, probably due to changes of shape. The comparison with the ephemeris shows a maximum displacement of 16<sup>m</sup>, or more than 3" at mean distance." The apparently irregular movement of the markings on Jupiter (to whatever cause it may be due) is a fact well known to all students of the planet, and it is highly probable (if indeed it was not sufficiently proved by the observations in 1903) that the same thing occurs on Saturn.

I hope to return to this subject with greater detail, but fear there is little prospect of unanimity between Professor Hough and other observers so long as he holds his present views on the various methods of taking transits.

Bishopston, Bristol: 1904 May 20.

Note on the Gyroscopic Collimator of Admiral Fleuriais. By M. E. J. Gheury.

Admiral Fleuriais's Gyroscopic Collimator is not by any means new, and although it has been greatly improved lately by Messrs. Ponthus and Therrode, of Paris, who kindly lent me one of their latest patterns, these improvements have left unaltered the characteristic features of the original collimator, being chiefly concerned in making the apparatus practical and easy in working, while in the hand of a skilful observer the results obtained with the latest form are of remarkable accuracy.

However, although a very interesting and elegant solution of the problem that sprung up with the requirements of modern navigation, the importance of which cannot be overrated, it is, I find, very little known either by seamen or by non-professional people interested in science generally. This is a pity, seeing the ingenuity displayed in overcoming the many obstacles preventing the realisation of a reliable artificial horizon. Observations at sea differ from those on land in this respect: while on land the utmost accuracy is but a question of meteorological conditions, at sea considerations of a mechanical nature, even with the most favourable weather, prevent the obtainment of more than an approximation. Indeed, a greater exactitude would not be greatly beneficial; one cannot be certain of the speed of a tide current more closely than within half a mile or so—even a good deal of experience is needed to ascertain it as closely—and no refinement in navigating methods would justify the captain of a ship to pass a rock at a distance of but a hundred yards, on pretence that he knows his position with mathematical accuracy. Where rocks are to be found tidal streams generally prevail, and in the best conditions a quarter of a mile is the closest berth the ablest navigator dares to give them.

Taking into consideration the various factors that enter into the determination of the position of a ship at sea, it may safely be said that to know that this position is within a circle on the chart of half a mile radius, or, to be more correct, and keeping in view the coordinate nature of the data, within a square the side of which is a nautical mile, is all that is required, and it is rare to obtain with certainty a smaller area of position, except in the neighbourhood of land, in calm, clear weather, by station-pointer

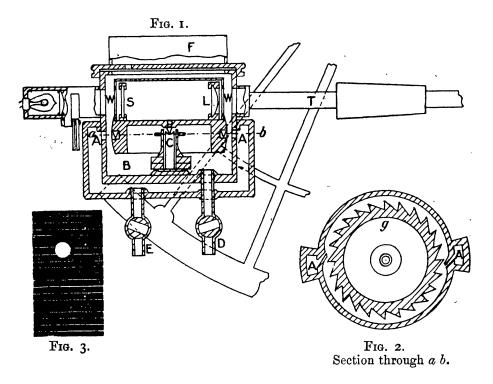
observations of some selected landmark.

The sea horizon gives the above approximation in fine weather, when there is no abnormal refraction—a fact of which one is not generally aware when taking the observation. It fails entirely when the reflection of the Sun falls on the sea line, in misty weather, and at night, except in few special cases, it is at best undefined. Although whenever it is available it answers all the requirements of navigation, in many cases the opportunity of a good "fix" allowed by a clear day sky or a starry night is lost or gives but a very rough indication of position, because of the want of a reliable horizon.

I shall not refer to the various appliances that can be used to obtain this line of reference, of paramount importance to the navigator. I have tried several of them, and found them to be either unreliable or to need such preparations and favourable state of the weather as to be unpractical. Amongst the unsuccessful attempts to solve the problem I will, however, mention two, which have a special interest with regard to the Gyroscopic One, invented, I believe, some sixty years ago, consisted in a horizontal mirror fixed on the top of a revolving gyroscope. The very nature of the support was the cause of failure, as the axis of such a revolving body is rarely vertical. The other was a pendulum, altitudes being taken with regard to a horizontal line marked on the bob when the latter was at three consecutive positions—at the beginning, middle, and end of a double oscillation. The altitude was then calculated for the mean position of the mark. This may be realised with a fixed point of suspension, but at sea such a point is nowhere to be found, and the forced oscillations introduced by the rolling of the ship had such a large influence as to entail a prohibitive length for the pendulum in order to have a period of oscillation sufficiently large compared with that of the rolling of the vessel not to be disturbed by the motion of the latter.

In the Gyroscopic Collimator the motion of precession, which was the cause of failure of the first type I just mentioned, is taken advantage of to obtain a pendulum of very small size, the time of oscillation of which is very large, by the similarity existing between the motion of a pendulum and that of the axis of the gyroscope on each side of a vertical plane through the pivot.

This instrument is used in connection with an ordinary sextant, the observation being taken, as with the sea horizon, by bringing the image of the observed body in a field of vision in which a horizontal grating of a special kind allows the observer to ascertain the position of the sensible horizon and the angular distance the image is from it.



At the centre of an air-tight case B (fig. 1) is fixed a small hemispherical cup C of hard material, the diameter of which is '1". This supports the pivot of the gyroscope, the latter having the shape of a spherical segment, the centre of which is coincident with the point of the pivot, while its centre of gravity is a little below this point, about '02". This segment is hollow, to combine lightness with a large moment of inertia with regard to the axis. Along its equator, in a groove V, are placed little vanes (fig. 2), by means of which the gyroscope is set in motion

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by the inflow of air from the apertures A in communication with the atmosphere at E, when air is pumped out of the case B by the tube D.

On the top of the gyroscope is the optical apparatus giving the sensible horizon. This consists in a plano-convex lens L and a field grating S, placed at the opposite extremities of a same diameter, the grating, which is a succession of equidistant transparent lines perpendicular to the axis of rotation, on a black

background (fig. 3), being at the focus of the lens.

The axis of this optical system is in the same horizontal plane as the line of sight of the observing telescope, the case B having two windows W facing each other along the line of sight, so that the light of an electric lamp, or of the Sun—reflected, in the latter case, in a small mirror fixed on the case—can pass through the latter and be received by the telescope whenever the axis of the optical system fixed on the gyroscope is coincident with the vertical plane through the collimation line of the telescope; this occurs at every half-revolution. As the normal speed is about 100 revolutions per second a persistent image of the grating is observed in bright lines on a dark background, on which the image of the observed body can be brought down.

When a top so supported is made to revolve round its axis, this axis describes a cone of decreasing amplitude, and a point on it will describe a closing spiral. If the eye is at about the level of the pivot, and in a fixed vertical plane through it, the axis will be seen to oscillate on each side of the vertical, exactly as a pendulum, the amplitude of the oscillations decreasing gradually. By decreasing the distance of the centre of gravity from the pivot, or increasing the speed of rotation, the time of a cycle of precession can be increased, so as to minimise the influence of the motion of the vessel. We have thus a short pendulum with the time of oscillation of a very long one. This motion of precession is really made up of a succession of minute loops, similar to the loops described by the Earth in its nutation, but, with a sufficient speed of rotation, these loops are too small to be detected by the eye, as it is the point of the pivot, and not the centre of gravity, which describes them.

When the axis of the gyroscope is vertical the middle of the dark space of the grating is the trace of the sensible horizon in the field, a constant instrumental error  $\pm e$ , made up of two parts, (a) the instrumental error of the sextant itself, and (b) the collimation error of the gyroscope, being allowed for. The axis, however, is rarely vertical, and its spiral motion will give a balancing motion to the lines of the grating. When the axis of rotation is in the plane of the sextant the lines are horizontal and parallel to a spider thread in the telescope, perpendicular to the plane of the sextant, the centre line being below or above the plane of the sensible horizon, according to the position of the axis, inclined towards or away from the observer. As the gyroscope rotates the lines become slightly inclined to the horizontal,

becoming horizontal again after half a cycle, and so on. If we only consider the grating when its lines are horizontal there is then, at the successive positions of the gyroscope at which this occurs, a vertical oscillation of the lines, while the reflected image of the observed body will be comparatively steady, and will seem therefore to oscillate vertically with regard to the grating. A little spring, not shown in the figure, and acted upon by a lever, allows a slight pressure to be exerted upon the gyroscope to give it the necessary inclination for this precessing motion.

When the gyroscope has attained its full speed the tube E is closed, and two or three strokes of the pump are given, so as to produce a certain vacuum (about 6 cms. of mercury), shown by the gauge F fixed on the top of the case B. The tube D is then closed, the pump disconnected, and the instrument is available for observations for a lapse of time of about half an hour.

The image of the observed celestial body being brought down on the image of the grating, its position with regard to the centre line is observed at each reversal of its motion, when the grating is horizontal. A little practice will enable the observer to estimate the tenths of the intervals between the consecutive lines. The time corresponding to each separate observation of the image is taken by an assistant.

Means of series of observations made upon a collimator, to study the behaviour of the instrument, have given evidence of systematic errors, varying with the "launching" without apparent reasons, and giving a maximum deviation of 2' from the general mean. In good conditions the errors of individual observations differ but slightly from the mean of each series. This is sufficient to enable the instrument to be very useful at sea. The wear and tear of the cup supporting the pivot seem to be the principal causes of the observed anomalies.

On fig. 3 we see the position of the lower limb of the Sun to be -50', each space being 10', and the telescope having an astronomical eye-piece. The positions of the star, from top to bottom, are -71', -46', and +34'. The space between the lines is not always 10', but is sometimes a little more or less; the readings must be therefore multiplied by a constant to get the true angular distance.\*

There are several ways of taking an observation. A first method is to observe three successive positions at which the observed body changes its motion—that is, a maximum included between two minima or two maxima on each side of a minimum. Then if a', a'', a''' are the three positions in chronological order they are respectively on fig. 3-71, +34, -46, and the correct reading a is

$$a = \frac{1}{4}(a' + 2a'' + a''')$$
 or here  $\frac{1}{4}(-71 + 68 - 46) = -12'$ .

\* For the instrument I have on trial the line space is 11' 30", the reducing constant being 1.15.

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From these three readings and the time of a cycle of precession, given by the interval between them, a correction I shall consider later on, due to the motion of the Earth, is calculated, together with the altitude corresponding to the motion of the gyroscope when the axis is vertical. In other words, the effect of the rotation of the Earth and of the erecting tendency of the gyroscope must be ascertained.

To observe well-defined maxima and minima the precession must have a certain minimum value, especially for the observation of a celestial body out of the meridian, the movement in altitude of which may attain 15" per second of time when on the prime vertical. Another method is to take a succession of readings as near each other as possible, either when the precession has almost ceased or with the full precession, as for the maxima and minima method. These, plotted with the time, will give in the first case almost a straight line from which the readings corresponding to any time may be taken; and, in the second, a wavy line, on which the maxima and minima can be selected and the corresponding times taken from the curve.

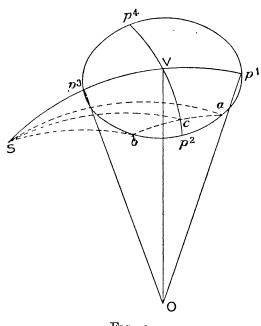
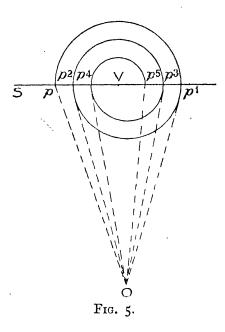


Fig 4.

When the celestial body is motionless, as it is approximately in a meridian observation, we can either take the mean of the three readings, and then we have Obs. Alt.  $A = Ai + \frac{1}{4}(a' + 2a'' + a''')$ , where Ai is the reading on the limb corrected for instrumental error—that is, Ai = Ao + e, Ao being the reading on the limb—or we can apply to any of the three readings a correction to bring it to what it would have been had the axis of the gyroscope been vertical by calculating the coefficient k of tendency to

verticality of the axis. In an observation out of the meridian, if the motion of the celestial body S in its vertical is larger than the motion of translation of the pole P of the gyroscope, the distance from S to P will increase continuously, and it will be impossible to have maxima and minima. Only if the translation of the pole is greater than the motion of the celestial body will the observation be possible. The theory of the instrument is rather long and complicated.\*

If the translation of the pole P is greater than that of the celestial body, from  $p_1$  to a the distance SP will increase, as at  $p_1$  the speed of approach of P towards S is zero; if a and b are the points for which the translation of the pole projected on the diameter  $p_1p_3$  is equal to the motion of the body on its vertical, at a and b SP will be constant, between a and b SP will decrease, and on the rest of the path  $bp_3p_4p_1a$  SP will increase again (fig. 4). The greater the speed of P compared to that of S the nearer a and b will be to  $p_1$  and  $p_3$ ; if  $ap_1$  and  $bp_3$  are very small,  $\frac{1}{2}(Sa+Sb) = Sc$  will be equal to SV nearly.



The speed of P is proportional to the radius of the path of precession—that is, to the inclination of the axis—the greater this inclination, the less will be  $ap_1$  and  $bp_3$ . It can be shown that a minimum inclination of 1° will be quite sufficient if the minimum permissible error in SV be 30".

If the point of the pivot is hemispherical the locus of a point on the axis is a loxodrome of the sphere:  $\frac{\nabla p}{pp_1} = \frac{\nabla p_1}{p_1p_2} =$ 

\* See the reports on the instrument published in the Annales Hydro-graphiques and the Revue Coloniale, of which what follows is a summary.

 $\frac{\nabla p_2}{p_2p_3} = k = \text{constant}$  (as long as the pivot does not alter in shape). In this expression  $k = \cdot 5 + E$ , E being proportional to  $\frac{Cr}{d}$ , where C = constant dependent upon the state of surfaces, r = radius of the spherical point, d = distance of the point from the centre of gravity (fig. 5).

The greater the radius of curvature of the point the greater k, the greater the speed with which the axis becomes vertical; k is therefore a coefficient of tendency to verticality. With a mathematically pointed pivot  $k = \cdot 5$  and the loxodrome becomes a circumference. All is therefore as if the positions Op,  $Op_1$ ,  $Op_2$ ... were the directions of a simple pendulum oscillating in

the plane OVS.

But  $SV = \frac{1}{2}(Sp + Sp_1)$  is not accurate; we have

$$SV = Sp + pV = Sp + kpp1,$$
  

$$SV = Sp1 - Vp1 = Sp1 - kp1p2,$$
  

$$SV = Sp2 + Vp2 = Sp2 + kp2p3,$$

Meridian Observation.—If the body is on the meridian three observations are sufficient to find k, for we have from the three equations above  $\mathrm{S}p_1 - \mathrm{S}p = k(pp_1 + p_1p_2)$ , and  $k = \frac{pp_1}{pp_1 + p_1p_2}$ . Let  $pp_1 = \mathrm{M}$ ,  $p_1p_2 = \mathrm{N}$ , then  $k = \frac{\mathrm{M}}{\mathrm{N} + \mathrm{M}}$ , M and N being the amplitudes of the first and second oscillations respectively. For the three positions of the star shown (fig. 3)  $k = \frac{139}{139 + 114} = .55$ . If  $\mathrm{A}i$  is the reading on the limb,  $s_1$ ,  $s_2$  two successive readings on the grating,  $s_2$  the reading that would have been made if the axis had been vertical, then the calculated altitude is  $\mathrm{A}c = \mathrm{A}i + sx$ . When the first reading  $s_1$  was taken  $\mathrm{S}$  was  $s_1$  from the central line, which was  $p_1 = kpp_1 = \mathrm{M}k$  from the horizon, and so  $s_2 = s_1 \pm \mathrm{M}k$ , then

$$Ac = Ai + si \pm Mk$$
, and similarly  $Ac = Ai + s2 \pm Nk$  . . .

But sx is between s1 and s2; no error on the sign of M or N is possible, it is always the opposite sign to s1 for M or s2 for N.

The reading on the limb Ai has, of course, been corrected for the instrumental error: this is complex, being partly due to want of parallelism between the reflecting mirrors of the sextant, to want of perpendicularity of these with regard to the plane of the instrument, and to the collimation error of the gyroscope itself, the latter error being due to the fact that the centre of the grating is not always exactly in the plane of the sensible horizon, even when the axis is vertical. Another correction in

the form of a multiplying factor to the readings on the grating is necessary, when the space between the lines is not 10' but, owing to mechanical inaccuracy, more or less. The correcting constants are ascertained either by observing an object of known altitude or by pointing on a collimator.

Observations out of the Meridian.—When the body is out of the meridian, one of the data of the observation is the time corresponding to a given altitude. This is obtained by time observations taken by an assistant at the required moment. If  $s_1$  and  $s_2$  are two consecutive readings,  $t_1$ ,  $t_2$  the corresponding times, s'' what the second reading would be if the body were fixed, then  $s_2 = s'' + dh(t_2 - t_1)$ , where dh is the motion in altitude of the body.

$$Ac = Ai + sI + (s'' + dh(tz - tI) - sI)k$$
  

$$Ac = Ai + sI + (s'' - sI)k + dh(tz - tI)k.$$

The calculated altitude is the altitude at time  $t_1$  plus the motion in altitude during the time  $(t_2-t_1)k$ , and the corresponding time is  $t_1+(t_2-t_1)k$ . We have, therefore,  $Ac=Ai+s_1+(s_2-s_1)k$ ,  $To=t_1+(t_2-t_1)k$ . If k is affected by an error dk, the first altitude obtained by  $s_1$  and  $s_2$  will be affected by an error dAi=+Mdk, and the second, given by  $s_2$  and  $s_3$ , by an error dAi=-Ndk; the mean altitude will be affected by an error (M-N)dk. It is, therefore, an advantage to take three readings, and to take the mean of the two altitudes. The experience shows that dk is generally not much greater than  $c_2$ . The error of altitude is then about  $c_1'$ .

To get the true altitude we have then

A =  $Ai + \sigma s_1 + \sigma k(s_2 - s_1) \pm e \pm i$ —refraction— $\frac{1}{2}$  diameter, where e is the instrumental correction for the sextant and the gyroscope combined;

i is the correction, to be considered later, of the influence of

the motion of the earth.

 $\sigma$  is the correcting factor due to the interline space not

being 10'.

The above method is open to the objection that the difference between the approximate altitude Ai and the calculated one Ac is very large in  $Ac = Ai + k(s_2 - s_1)$ . The calculation of  $k = \frac{M}{M+N}$  is also rather long.

A method of calculation due to Commandant Guyou overcomes these defects, and is worth mentioning.

Meridian Observation.—The above process is not rational, as it gives to three readings the weight of which is the same widely different functions. A first simplification is to take as approximate reading the mean of the two first readings, as follows: when  $s_1$  is taken, the mean line of the grating is distant from the sensible horizon by  $pV = pp_1k = Mk$ ;  $s_1$  referred to the sensible horizon is therefore  $s'_x = s_1 + Mk$ , and  $s_2$  referred to that horizon

is  $s''_x = s2 \pm Nk$ . The mean will become  $s_x = \frac{1}{2}(s'_x + s''_x) = \frac{1}{2}(s_1 \pm Mk + s_2 \pm Nk) = \frac{1}{2}(s_1 - s_2) + k\frac{M - N}{2} = \frac{s_1 + s_2}{2} + \frac{M(M - N)}{2(M + N)}$ , but  $\frac{1}{2}M = \frac{1}{2}pp_1 = \frac{1}{2}(Sp_1 - Sp_2) = \frac{1}{2}(s_2 - s_1)$ , so that

$$s_x = \frac{1}{2}(s_1 + s_2) - (M - N)\frac{\frac{1}{2}(s_2 - s_1)}{M + N}.$$

Let  $\frac{M-N}{M+N}$ =R, then  $s_x = \frac{1}{2}(s_1+s_2)+\frac{1}{2}R(s_2-s_1)$ ; the correction

is much smaller. The mean of the means is  $\frac{1}{2} \left( \frac{s_1 + s_2}{2} + \frac{s_2 + s_3}{2} \right)$ ; then

$$s_x = \frac{1}{2} \left( \frac{s_1 + s_2}{2} + \frac{s_2 + s_3}{2} \right) + \frac{1}{2} R \left( \frac{s_2 - s_1}{2} - \frac{s_2 - s_3}{2} \right).$$

The correction is smaller still; it may be written

$$\frac{M-N}{2(M+N)} \left(\frac{pp_1}{2} - \frac{p_1p_2}{2}\right) = \frac{M-N}{2(M+N)} \left(\frac{1}{2}M - \frac{1}{2}N\right), \text{ and we have}$$

$$s_x = \frac{1}{2} \left(\frac{s_1 + s_2}{2} + \frac{s_2 + s_3}{2}\right) + \frac{(M-N)^2}{4(M+N)}.$$

The correction being very small, a set of curves gives its value with a sufficient exactitude. (See *Annales Hydrographiques* for 1901.)

Observations out of the Meridian.—It is preferable to reduce all the observations to the same instant by correcting for the motion in altitude of the celestial body:  $dh = \frac{1}{4} \sin Z \cos L dT$ , and the case is simplified into the preceding one, where the body has no motion along its vertical. The calculation is long, but is simplified by two tables due to Enseigne de Vaisseau Cretin (Annales Hydrographiques, 1901, pp. 96 and 98). Table I. gives in seconds of arc the motion of a body in altitude for each second of time:  $dh = 15 \sin Z \cos L$ , argument: Z and L. Table II. gives, in function of the result found in Table I., the movement in altitude for an interval of 30 to 74 seconds, argument: T and dh from Table I.

Influence of the Motion of the Earth.—As may be expected the motion of the gyroscope is perceptibly influenced by the motion of the Earth. The direction of the motion of precession depends on the direction of rotation of the gyroscope and on the position of the pivot with regard to the centre of gravity. The rotation being clockwise, looking from above the instrument, the precession is counter-clockwise.

The effect of the rotation of the Earth is to make the axis of the gyroscope precess round a deviated vertical, the true zenith being then between the apparent zenith and the celestial pole. For our hemisphere the altitudes observed, facing north, will be always too small, those facing south always too large, the sign of the correction depending of course on the direction of rotation of the gyroscope. At the pole, for L = 90°, and when the observed body is due east or west, the correction is zero. It is maximum for meridian observations, being  $\sin i = \frac{2\text{T cos L}}{86400}$  (fig. 6). Out of the meridian in S the correction is  $ZE = Z'S - ZS = ZZ'\cos Z$  approximately, Z being reckoned from the north pole always. The correction is, therefore,  $i' = \frac{2\text{T cos } Z \cos L}{25}$ , its sign depending on the sign of Z and L.

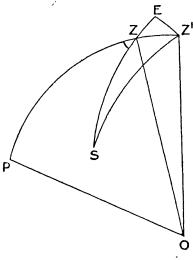


Fig. 6.

The correction being proportional to the time of precession, this time must be known. In the method by maxima and minima it is given by the interval between two consecutive maxima or minima. However, when the method of damped precession is resorted to this time cannot be ascertained by observation. For a small inclination of the axis with regard to the vertical, and for a theoretically sharp pivot, the speed of precession  $V = \frac{gd}{r^2w}$ , where g = the gravitation constant, r = the radius of gyration, and w = the angular speed of rotation. If T is the time taken to perform half a turn of precession,  $V = \frac{2\pi}{2T} = \frac{gd}{2\pi Nr^2}$ , where N is the revs. per second and d the distance of the point Then  $T = \frac{2\pi^2 r^2 N}{gd} = \frac{2r^2 N}{d}$  nearly, from the centre of gravity. writing  $\pi^2 = g$ . The time of precession is therefore independent of the mass and inclination of the gyroscope. The correction becomes then, if  $c = \frac{4r^2N}{25d}$ ,

$$i' = \frac{4r^2N\cos Z\cos L}{25d} = c\cos Z\cos L.$$

The number of turns can only be ascertained by careful stroboscopic measurements, and this being a delicate operation it would be impossible to perform it as an auxiliary observation when taking the altitudes. A lengthy series of observations was therefore made in 1902 and 1903 by M. L. Favé, at the Service Hydrographique, to ascertain how far the theory was able to give formulæ from which the number of turns could be deduced.

The mechanical constants necessary for these investigations were obtained by oscillating the gyroscope at rest. The gyroscope is really a compound pendulum, and its oscillations are of a somewhat complicated nature, being plane but for a very short time and soon becoming conical. The axis then describes an elliptic cone, the major axis of which rotates in azimuth, the eccentricity decreasing till, after the major axis has turned 45°, the ellipse has become a circle; after this it becomes an ellipse again. After a while the oscillation is again in a plane perpendicular to the primitive plane, and the same series of motions reproduces itself in reverse order till the oscillation is again in the primitive plane. This is due to the fact that the body is not entirely of revolution; moreover the pivot is not theoretically sharp. The determination of the time of oscillation is therefore a delicate operation.

We have then

$$\theta = \pi \sqrt{\frac{d + \frac{B}{md}}{g}}$$

where

 $\theta$  is the time of a simple oscillation in seconds,

d is the distance of the point of the pivot to the c. g.,

B is the moment of inertia with regard to a perpendicular to the axis of figure passing through the c. g.;

m = w/g is the mass in grams.

Two values of  $\theta$  were sufficient to obtain B and d. From B the value of the radius of gyration r was obtained; the value of c could therefore be calculated for any value of N.

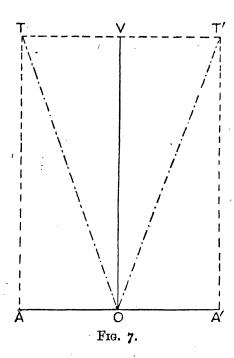
Measurements were also taken with a stroboscope at different times, so as to give the decrease of N corresponding to the time elapsed since the launching of the gyroscope. This was done for various degrees of vacuum, and a set of curves was obtained (see Annales Hydrographiques for 1904). The time of half a cycle of precession being ascertained at the beginning, when the precession can be easily observed, the time elapsed between this observation and the astronomical observation gives, on the curve corresponding to the vacuum indicated by the gauge on the instrument, the number of turns at the time the altitude was observed, or, by means of a special scale, c directly.

It was found, however, that, if the stroboscope is not used, the approximation for the correction of the motion of the Earth is but 20 per cent., or about 1' in the most unfavourable case.

The next improvement needed seems, therefore, a method to

measure N at the time of the observation.

The Tables I. and II. mentioned above may also be used for the correction of the Earth's motion. For  $i' = \frac{2T\cos Z\cos L}{25}$ ,  $i' = \frac{T}{3} \frac{15\cos Z\cos L}{60}$ , nearly. Table I. gives 15 sin Z cos L; if it is entered with 90°—Z it gives 15 cos Z cos L. Table II. gives the product of T by this number from Table I. reduced to minutes—that is,  $\frac{15T\cos Z\cos L}{60}$ . Table II. gives therefore a number equal to three times the correction.



Influence of the Motion of the Ship.—The motion of the ship communicates to the pivot O horizontal accelerations OA, OA'; OV being the vertical, the axis precesses around it. If gravitation ceased to act it would precess around OA. The motion of the ship makes it to precess around the resultant OT. The pole will oscillate from T to T' according to the direction of acceleration (fig. 7).

The rolling motion from one extreme position on one side to that on the other side, if the total time of a single oscillation be t, consists in three phases of approximately equal duration t/3, the first being one of uniformly accelerated motion; the second of uniform motion, the last of uniformly retarded motion.

If the motion of the ship was uniform the radius  $\nabla p$  would describe around the vertical 3° in 1 sec. about, if 2T is equal to two minutes. The rolling of the ship will bring the pole successively in T, V, T' . . . V being a point on the true vertical and T, T' points on the deviated vertical (fig. 8).

Suppose a rolling period of six secs. for a single oscillation, let the acceleration be in the direction VT, the pole will precess round T for two seconds and will reach 1, the next two seconds (period of uniform motion) it will precess round V and reach 2,

then round T' and reach 3... &c.

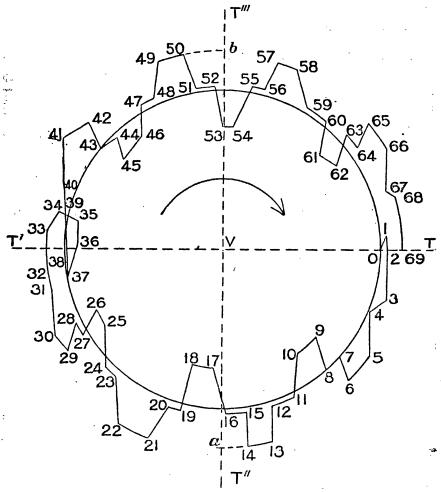


Fig. 8. Influence of the Motion of the Vessel. (Precession 2<sup>m</sup> 18<sup>s</sup>, clockwise.)

We see that in the direction TT' the path of the pole differs little from the circle it would follow if the ship were steady. In the direction T"T" the deviation is considerable. In this direction, if the ship were motionless, the central line of the image of the grating would rise above the sensible horizon and descend below it in a quantity corresponding to VO. When the ship is

rolling it will ascend in the field to, say, the position 50, stop from 53 to 54, and rise again till 57, after which the descent will be more marked. After b the same thing will occur and the centre of the image of the grating will attain its lower position at 13;  $V_1a$ ,  $V_2b$  are about equal, the middle of the maximum angle coincides approximately with the trace of the sensible horizon. If one observes in the direction perpendicular to the swell—that is, in the direction of the acceleration, as on the beam of a rolling ship—the error will be very small. In the perpendicular direction only the extreme positions on the grating must be observed, and one must be careful not to observe false maxima or minima.

In practice Messrs. Arago, Baule, Boyer, and Schwerer have found that in small boats the error is always negligible, in ships at anchor it is negligible if the extreme positions are observed, on a ship moving with moderate speed in moderate weather the maximum error is less than 3', and on a fast ship in a heavy sea (this practically never occurring in a fog) the error may be 5' and more.

This drawback will perhaps be considered serious, but it detracts but little from the utility of the instrument, for the occasions when in heavy weather the sea horizon is not visible are exceedingly rare. At night, if the stars are out, they may be selected in a favourable direction, and even if this cannot be done the course of the ship can be altered for few minutes while the observation is taken, as is often done with far less necessity to ascertain the deviation of the compass on a course different to that which the ship is following.

The gyroscopic collimator (or gyroscopic horizon) of Admiral Fleuriais seems, so far, to be the instrument which, at sea, is likely to give results that, while they are at least as accurate as (and, if the method of a series of observations in rapid succession is resorted to, much more accurate than) those obtained with any other artificial horizon, are obtained in such an easy and convenient way that it cannot be compared with any other instrument used for the same purpose. All the necessary preparations can be made under shelter, in the chart room, and on the observer setting his foot on deck the instrument is at once ready for observing. Any one who has tried to take observations with an artificial horizon in actual sea-going weather and conditions, and not merely to kill time on a fine sunny day, will welcome the instrument as a great step towards the possibility of observing in any weather when there is "something out" to observe.

A certain skill is needed to obtain good results, and care must be taken in handling the instrument, but a man in command of a large modern steamer is not likely to lack in the necessary ability.

The cost of the complete apparatus, octant, gyroscope, pump, accumulators, and spare pieces, is 26l. The total weight of the instrument is  $4\frac{1}{2}$  lbs.